

# Astronomical constraints on some long-range models of modified gravity

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## Abstract

In this paper we use the corrections to the usual Newton-Einstein secular precessions of the perihelia of the inner planets of the Solar System, phenomenologically estimated as solve-for parameters by the Russian astronomer E.V. Pitjeva by fitting almost one century of data with the EPM2004 ephemerides, in order to constrain some long-range models of modified gravity recently put forth to address the dark energy and dark matter problems. The models examined here are the four-dimensional ones obtained with the addition of inverse powers and logarithm of some curvature invariants, and the multidimensional braneworld model by Dvali, Gabadadze and Porrati (DGP). After working out the analytical expressions of the secular perihelion precessions induced by the corrections to the Newtonian potential of such models, we compare them to the estimated corrections to the rates of perihelia by taking their ratio for different pairs of planets instead of using one perihelion at a time for each planet separately, as done so far in literature. As a result, the curvature invariants-based models are ruled out, even by re-scaling by a factor 10 the errors in the planetary orbital parameters estimated by Pitjeva. Less neat is the situation for the DGP model. Only the general relativistic Lense-Thirring effect, not included, as the other exotic models considered here, by Pitjeva in the dynamical force models used in the estimation process, passes such a test. It would be important to repeat the present analysis by using corrections to the precessions of perihelia independently estimated by other teams of astronomers as well, but, at present, such rates are not yet available.

Keywords: Experimental tests of gravitational theories; Modified theories of gravity; Celestial mechanics; Orbit determination and improvement; Ephemerides, almanacs, and calendars

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# 1 Introduction

Weak-field limits of theories involving long-range modifications of gravity recently put forth to address the issues of dark energy and dark matter (Dvali et al., 2000; Allemandi et al., 2005; Navarro and van Acoleyen, 2005, 2006a,b; Apostolopoulos and Tetradis, 2006; Capozziello et al., 2006; Nojiri and Odintsov, 2007) are important because such exotic corrections to the Newtonian potential allow, in principle, for tests to be performed on local, astronomical scales, independently of the galactic/cosmological effects (Capozziello, 2007) which motivated such alternative theories and that, otherwise, would represent their only justification. In this paper we will show how to obtain phenomenologically tight constraints on the viability of some of such modified theories by suitably using the latest observational results from Solar System planetary motions (Pitjeva, 2005a,b).

In Section 2 we will consider the four-dimensional models obtained from inverse powers of some curvature invariants (Navarro and van Acoleyen, 2005). After working out the analytic expression of the secular, i.e. averaged over one orbital revolution, perihelion precession induced by the Newtonian limit of such models, we will compare it to the phenomenologically estimated corrections to the usual Newton-Einstein precessions of the perihelia of the inner planets of the Solar System. By taking the ratio of them for different pairs of planets we will find that the predicted exotic effects are ruled out. In Section 3 and Section 4 we repeat the same procedure for the four-dimensional model based on the logarithm of some invariants of curvature (Navarro and van Acoleyen, 2006b) and for the multidimensional braneworld model by Dvali et al. (2000), respectively, by finding that also such models do not pass our test. In Section 5 we apply the same strategy to the general relativistic gravitomagnetic field (Lense and Thirring, 1918) finding that it is, instead, compatible with the ratio of the perihelion precessions for all the considered pairs of planets. Section 6 is devoted to the conclusions.

## 2 The inverse-power curvature invariants models

In this Section we will address the long-range modifications of gravity obtained by including in the action inverse powers of some invariants of curvature not vanishing in the Schwarzschild solution (Navarro and van Acoleyen, 2005, 2006a,b). From the correction to the Newtonian potential (Navarro

and van Acoleyen, 2005)

$$V = \frac{\alpha GM}{r_c^{6k+4}} r^{6k+3}, \quad r \ll r_c, \quad (1)$$

where  $k$  is a positive integer number, it follows a purely radial acceleration

$$\mathbf{A} = -\frac{\alpha GM(6k+3)}{r_c^{6k+4}} r^{6k+2} \hat{\mathbf{r}}, \quad r \ll r_c. \quad (2)$$

The length scale  $r_c$  depends, among other things, on a parameter  $\mu$  which must assume a certain value in order that the model by Navarro and van Acoleyen (2005) is able to reproduce the cosmic acceleration (Carroll et al., 2005; Mena et al., 2006) without dark energy; it is just such a value of  $\mu$  which makes  $r_c \approx 10$  pc ( $k = 1$ ) for a Sun-like star (Navarro and van Acoleyen, 2005). Since (Navarro and van Acoleyen, 2006a)

$$\alpha = \frac{k(1+k)}{(6k+3)2^{4k}3^k} \quad (3)$$

and  $r_c \approx 10$  pc ( $k = 1$ ), the condition  $r \ll r_c$  for which the expansion in  $r/r_c$  yielding eq. (1) retains its validity is fully satisfied in the Solar System, and eq. (2) can be treated as a small correction to the Newtonian monopole with the standard perturbative techniques of celestial mechanics. The Gauss equation for the variation of the pericentre  $\omega$  of a test particle acted upon by an entirely radial disturbing acceleration  $A$  is

$$\frac{d\omega}{dt} = -\frac{\sqrt{1-e^2}}{nae} A \cos f, \quad (4)$$

where  $a$  and  $e$  are the semimajor axis and the eccentricity, respectively, of the orbit of the test particle,  $n = \sqrt{GM/a^3}$  is the unperturbed Keplerian mean motion and  $f$  is the true anomaly reckoned from the pericentre. The secular precession of the pericentre  $\langle \dot{\omega} \rangle$  can be worked out by evaluating the right-hand side of eq. (4) onto the unperturbed Keplerian ellipse

$$r = a(1 - e \cos E), \quad (5)$$

where  $E$  is the eccentric anomaly, and by performing subsequently an integration over one full orbital period. To this aim, the following relations are useful

$$dt = \left( \frac{1 - e \cos E}{n} \right) dE, \quad (6)$$

$$\cos f = \frac{\cos E - e}{1 - e \cos E}. \quad (7)$$

Let us start with the case  $k = 1$ ; the extra-acceleration becomes

$$\mathbf{A}_{k=1} = -\frac{9\alpha GM}{r_c^{10}} r^8 \hat{\mathbf{r}}. \quad (8)$$

By proceeding as previously outlined and using the exact result

$$\int_0^{2\pi} (\cos E - e)(1 - e \cos E)^8 dE = -\frac{5e\pi}{64} [128 + 7e^2 (128 + 160e^2 + 40e^4 + e^6)], \quad (9)$$

it is possible to obtain the exact formula

$$\langle \dot{\omega} \rangle_{k=1} = -\frac{45\alpha}{r_c^{10}} \sqrt{GMa^{17}(1-e^2)} \left[ 1 + 7e^2 \left( 1 + \frac{5}{4}e^2 + \frac{5}{16}e^4 + \frac{1}{128}e^6 \right) \right]. \quad (10)$$

It is important to note the dependence of  $\langle \dot{\omega} \rangle$  on a positive power of the semimajor axis  $a$ : this fact will be crucial in setting our test.

The predicted extra-precession of eq. (10) can be fruitfully compared to the corrections to the usual Newton-Einstein perihelion rates of the inner planets of the Solar System phenomenologically estimated by Pitjeva (2005a), in a least-square sense, as solve-for parameters of a global solution in which a huge amount of modern planetary data of all types covering about one century were contrasted to the dynamical force models of the EPM2004 ephemerides (Pitjeva, 2005b). Such corrections are quoted in Table 1. They were determined in a model-independent way, without modeling this or that particular model of modified gravity: only known Newton-Einstein accelerations<sup>1</sup> were, in fact, modeled so that the estimated perihelion extra-rates account, in principle, for all the unmodeled forces present in Nature. Since July 2005 (Iorio, 2007a), many other authors so far used the extra-precessions of the perihelia of the inner planets of the Solar System estimated by Pitjeva (2005a) to put constraints on modified models of gravity (Gannouji et al., 2006; Iorio, 2006c; Sanders, 2006; Adkins and McDonnell, 2007; Ruggiero and Iorio, 2007; Iorio, 2007b), cosmological constant (Iorio, 2006a; Sereno and Jetzer, 2006b), various cosmological issues (Adkins et al., 2007; Fay et al., 2007; Nesseris and Perivolaropoulos, 2007a,b; Sereno and Jetzer, 2007), dark matter distribution (Iorio, 2006b; Khriplovich and Pitjeva, 2006; Sereno and Jetzer, 2006a; Frère et al., 2007;

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<sup>1</sup>With the exception of the general relativistic gravitomagnetic interaction, yielding the Lense-Thirring effect, and of the Kuiper Belt Objects.

Table 1: Semimajor axes  $a$ , in AU ( $1 \text{ AU} = 1.49597870691 \times 10^{11} \text{ m}$ ), and phenomenologically estimated corrections to the Newtonian-Einsteinian perihelion rates, in arcseconds per century ( $" \text{ cy}^{-1}$ ), of Mercury, the Earth and Mars (Pitjeva, 2005a). Also the associated errors are quoted: they are in m for  $a$  (Pitjeva, 2005b) and in  $" \text{ cy}^{-1}$  for  $\dot{\varpi}$  (Pitjeva, 2005a). For the semimajor axes they are the formal, statistical ones, while for the perihelia they are realistic in the sense that they were obtained from comparison of many different solutions with different sets of parameters and observations (Pitjeva, private communication 2005). The results presented in the text do not change if  $\delta a$  are re-scaled by a factor 10 in order to get more realistic uncertainties.

Planet	$a$ (AU)	$\delta a$ (m)	$\dot{\varpi}$ ( $" \text{ cy}^{-1}$ )	$\delta \dot{\varpi}$ ( $" \text{ cy}^{-1}$ )
Mercury	0.38709893	0.105	-0.0036	0.0050
Earth	1.00000011	0.146	-0.0002	0.0004
Mars	1.52366231	0.657	0.0001	0.0005

Khriplovich, 2007), trans-Neptunian bodies (Iorio, 2007c), general relativity (Will, 2006; Iorio, 2007a); a common feature of all such analyses is that they always used the perihelia separately for each planet, or combined linearly by assuming that the exotic effects investigated were included in the estimated corrections to the perihelia precessions, and by using their errors to constrain the parameters of the extra-forces. About the reliability of the results by Pitjeva (2005a), Iorio (2007b) made an independent check by assessing the total mass of the Kuiper Belt Objects and getting results compatible with other ones obtained with different methods, not based on the dynamics of the inner planets. It must be noted that more robustness could be reached if and when other teams of astronomers will estimate their own corrections to the perihelion precessions. On the other hand, an alternative approach would consist in re-fitting the entire data set by including an ad-hoc parameter accounting for just the exotic effect one is interested in. However, such a procedure might be not only quite time-consuming because of the need of modifying the software's routines by including the extra-accelerations, but it would be also model-dependent by, perhaps, introducing the temptation of more or less consciously tweaking somehow the data and/or the procedure in order to obtain just the outcome one a-priori expects.

Here we will not use one perihelion at a time for each planet. Indeed, let us consider a pair of planets A and B and take the ratio of their estimated

extra-rates of perihelia: if eq. (10) is responsible for them, then the quantity<sup>2</sup>

$$\Gamma_{AB} = \left| \frac{\dot{\omega}^A}{\dot{\omega}^B} - \left( \frac{a^A}{a^B} \right)^{17/2} \right| \quad (11)$$

must be compatible with zero, within the errors. The figures of Table 1 tell us that it is definitely not so: indeed, for A=Mars, B=Mercury we have

$$\Gamma_{\text{MaMe}} = 10^5 \pm 0.1. \quad (12)$$

The situation is slightly better for A=Mars and B=Earth:

$$\Gamma_{\text{MaE}} = 38 \pm 3.5. \quad (13)$$

An intermediate case occurs for A=Earth and B=Mercury:

$$\Gamma_{\text{EMe}} = 10^3 \pm 0.2. \quad (14)$$

It is important to note that

- The uncertainty in  $\Gamma_{AB}$  has been conservatively estimated as

$$\delta\Gamma_{AB} \leq \left| \frac{\dot{\omega}^A}{\dot{\omega}^B} \right| \left( \frac{\delta\dot{\omega}^A}{|\dot{\omega}^A|} + \frac{\delta\dot{\omega}^B}{|\dot{\omega}^B|} \right) + \frac{17}{2} \left( \frac{a^A}{a^B} \right)^{17/2} \left( \frac{\delta a^A}{a^A} + \frac{\delta a^B}{a^B} \right) \quad (15)$$

by linearly adding the individual terms coming from the propagation of the errors in  $\dot{\omega}$  and  $a$  in eq. (4); this is justified by the existing correlations among the estimated Keplerian orbital elements<sup>3</sup>

- The results presented here do not change if we re-scale by a factor 10 the formal errors in the semimajor axes (Pitjeva, 2005b) quoted in Table 1. The same holds also for the errors in the perihelia rates which, however, are not the mere statistical ones but are to be considered as realistic, as explained in the caption of Table 1
- The constraints obtained here with eq. (4) are independent of  $\alpha$  and  $r_c$ ; should one use eq. (10) for each planet separately to constrain  $r_c$ , it turns out that, for  $\alpha = 4 \times 10^{-3}$  ( $k = 1$ ),  $r_c \lesssim 4.5$  AU. Note that with such a value the condition  $r \ll r_c$ , with which eq. (1) and, thus, eq. (10) were derived, holds for all the inner planets
- For  $k > 1$  the situation is even worse because of the resulting higher powers with which the semimajor axis enters the formulas for the perihelion rates

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<sup>2</sup>It turns out that the multiplicative term depending on the eccentricities has a negligible effect on our conclusions.

<sup>3</sup>The correlations among the perihelion rates are low, with a maximum of 20% between the Earth and Mercury (Pitjeva, private communication, 2005).

### 3 The logarithmic curvature invariants models

The same approach can be fruitfully used for the model by [Navarro and van Acoleyen \(2006b\)](#) based on an action depending on the logarithm of some invariants of the curvature in order to obtain a modification of gravity at the MOND ([Milgrom, 1983](#)) characteristic scale ([Sanders and McGaugh, 2002](#)), so to address in a unified way the dark energy and dark matter problems; in this model the length scale  $r_c$  amounts to about 0.04 pc for the Sun. The correction to the Newtonian potential is

$$V \propto \frac{GM r^3}{r_c^4}, \quad (16)$$

which yields the perturbing acceleration

$$\mathbf{A} \propto \frac{r^2}{r_c^4} \hat{\mathbf{r}}. \quad (17)$$

By using

$$\int_0^{2\pi} (\cos E - e)(1 - e \cos E)^2 dE = -e\pi(4 + e^2), \quad (18)$$

the secular precession of perihelion induced by eq. (17) is

$$\langle \dot{\omega} \rangle \propto \frac{\sqrt{GM a^5 (1 - e^2)}}{r_c^4} (4 + e^2); \quad (19)$$

also in this case it depends on a positive power of the semimajor axis; cfr. the approximated result by [Navarro and van Acoleyen \(2006b\)](#) for the shift per orbit, i.e.  $2\pi \langle \dot{\omega} \rangle / n$ .

By taking the ratio of eq. (19) for a pair of planets and comparing it to the ratio of the estimated extra-precessions by [Pitjeva \(2005a\)](#) it can be obtained

$$\Delta_{AB} = \left| \frac{\dot{\omega}^A}{\dot{\omega}^B} - \left( \frac{a^A}{a^B} \right)^{5/2} \right|. \quad (20)$$

The test is not passed. Indeed, for A=Mars and B=Mercury we have

$$\Delta_{\text{MaMe}} = 30.7 \pm 0.1; \quad (21)$$

the pair A=Earth, B=Mercury yields

$$\Delta_{\text{EMe}} = 10.6 \pm 0.2, \quad (22)$$

while A=Mars, B=Earth  $\Delta$  is marginally compatible with zero

$$\Delta_{\text{MaE}} = 3.4 \pm 3.5. \quad (23)$$

Note that, even if the real errors in the estimated extra-precessions of perihelia were up to 10 times larger than those quoted by [Pitjeva \(2005a\)](#), the pair Mars-Mercury would still be able to rule out the logarithmic model by [Navarro and van Acoleyen \(2006b\)](#).

## 4 The multidimensional braneworld Dvali-Gabadadze-Porrati model

Another modified model of gravity aimed to explain the cosmic acceleration without dark matter is the multidimensional braneworld model DGP ([Dvali et al., 2000](#)) which predicts, among other things, an extra-rate of perihelion independent of the planetary semimajor axis<sup>4</sup> ([Lue and Starkman, 2003](#); [Iorio, 2005](#)). It is incompatible with the test of the ratio of perihelia as well, although less dramatically than the previously examined models. Indeed, by defining

$$\Psi_{\text{AB}} = \left| \frac{\dot{\omega}^{\text{A}}}{\dot{\omega}^{\text{B}}} - 1 \right|, \quad (24)$$

for A=Mars, B=Mercury we have

$$\Psi_{\text{MaMe}} = 1.0 \pm 0.2, \quad (25)$$

while A=Earth, B=Mercury yield

$$\Psi_{\text{EMe}} = 0.9 \pm 0.2. \quad (26)$$

Errors in the determined extra-rates of perihelion 5 times larger than those quoted in Table 1 would allow the DGP model to pass the test. The pair A=Mars, B=Earth give a result compatible with zero:

$$\Psi_{\text{MaE}} = 1.5 \pm 3.5; \quad (27)$$

the same hold for the other three combinations in which A and B denotes the planets with the smaller and larger semimajor axes, respectively. Until now, the DGP model was not found in disagreement with the Solar System data because the perihelia were used separately for each planet ([Iorio, 2006c](#)).

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<sup>4</sup>The only dependence on the features of the planetary orbits occurs through a correction quadratic in the eccentricity  $e$  ([Iorio, 2005](#)) which turns out to be negligible in this case.

## 5 General relativistic effects: gravitomagnetism and the cosmological constant

It maybe interesting to note that, contrary to the exotic effects induced by the modified models of gravity previously examined, the Lense-Thirring effect (Lense and Thirring, 1918) induced by the general relativistic gravitomagnetic field of the Sun, not modeled by Pitjeva (2005a), does pass our test based on the ratio of the perihelia. Indeed, since the Lense-Thirring perihelion precessions are proportional to a negative power of the semimajor axis, i.e.

$$\langle \dot{\omega} \rangle \propto a^{-3}, \quad (28)$$

the quantity

$$\Lambda_{AB} = \left| \frac{\dot{\omega}^A}{\dot{\omega}^B} - \left( \frac{a_B}{a_A} \right)^3 \right| \quad (29)$$

must be considered. It turns out that it is compatible with zero for all the six combinations which can be constructed with the data of Table 1. This result enforces the analysis by Iorio (2007a) in which the extra-rates of the perihelia were used one at a time for each planet and linearly combined by finding the general relativistic predictions for the Lense-Thirring precessions compatible with them.

## 6 Conclusions

In this paper we used the corrections to the Newton-Einstein secular precessions of the perihelia of the inner planets of the Solar System, estimated by Pitjeva (2005a) in a least-square sense as phenomenological solve-for parameters of a global solution in which almost one century of data were fitted with the EPM2004 ephemerides, to put tight constraints on several models of modified gravity recently proposed to explain dark energy/dark matter issues. By using the ratio of the perihelion precessions for different pair of planets, instead of taking one perihelion at a time for each planet as done so far, we were able to rule out all the considered long-range models of modified gravity, in particular the ones based on inverse powers of curvature invariants by Navarro and van Acoleyen (2005) and on the logarithm of some curvature invariants (Navarro and van Acoleyen, 2006b), even by re-scaling by a factor 10 the errors in the estimated perihelion extra-rates. The situation is less dramatic for the DGP (Dvali et al., 2000) braneworld model since if the real errors in the perihelion precessions were, in fact, 5 times larger

than the ones released it would become compatible with the data. Only the general relativistic Lense-Thirring effect passed the test. However, it must be noted that our results are based only on the extra-rates of perihelia determined by Pitjeva (2005a): it would be highly desirable to use corrections to the secular motion of perihelia estimated by other teams of astronomers as well. If and when they will be available our test will become more robust.

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## References

- Adkins, G.S., and McDonnell, J., Orbital precession due to central-force perturbations, *Phys. Rev. D*, **75**(8), 082001, 2007.
- Adkins, G.S., McDonnell, J., and Fell, R.N., Cosmological perturbations on local systems, *Phys. Rev. D*, **75**(6), 064011, 2007.
- Allemandi, G., Francaviglia, M., Ruggiero, M.L., and Tartaglia, A., Post-newtonian parameters from alternative theories of gravity, *Gen. Relativ. Gravit.*, **37**(11), 1891-1904, 2005.
- Apostolopoulos, P.S., and Tetradis, N., Late acceleration and  $w=-1$  crossing in induced gravity, *Phys. Rev. D*, **74**(6), 064021, 2006.
- Capozziello, S., Cardone, V.F., and Francaviglia, M.,  $f(R)$  theories of gravity in the Palatini approach matched with observations, *Gen. Relativ. Gravit.*, **38**(5), 711-734, 2006.
- Capozziello, S., Dark Energy Models Toward Observational Tests and Data, *Int. J. Geom. Meth. Mod. Phys.*, **4**(1), 53-78, 2007.
- Carroll, S.M., de Felice, A., Duvvuri, V., Easson, D.A., Trodden, M., and Turner, M.S., Cosmology of generalized modified gravity models, *Phys. Rev. D*, **71**(6), 063513, 2005.
- Dvali, G., Gabadadze, G., and Porrati, M., 4D gravity on a brane in 5D Minkowski space, *Phys. Lett. B*, **485**(1-3), 208-214, 2000.
- Fay, S., Nesseris, S., and Perivolaropoulos, L., Can  $f(R)$  Modified Gravity Theories Mimic a  $\Lambda$ CDM Cosmology?, *Phys. Rev. D*, at press, , arXiv:gr-qc/0703006v3, 2007.

- Frère, J.-M., Ling, G., and Vertongen, G., Bound on the Dark Matter Density in the Solar System from Planetary Motions, arXiv:astro-ph/0701542v1, 2007.
- Gannouji, R., Polarski, D., Ranquet, A., and Starobinski, A., Scalar tensor models of normal and phantom dark energy, *J. Cosmol. Astropart. Phys.*, **9**, 16, 2006.
- Iorio, L., On the effects of Dvali Gabadadze Porrati braneworld gravity on the orbital motion of a test particle, *Class. Quantum Grav.*, **22**(24), 5271-5281, 2005.
- Iorio, L., Can Solar System observations tell us something about the cosmological constant?, *Int. J. Mod. Phys. D*, **15**(4), 473-475, 2006a.
- Iorio, L., Solar System planetary orbital motions and dark matter, *J. Cosmol. Astropart. Phys.*, **5**, 2, 2006b.
- Iorio, L., On the perspectives of testing the Dvali-Gabadadze-Porrati gravity model with the outer planets of the Solar System, *J. Cosmol. Astropart. Phys.*, **8**, 7, 2006c.
- Iorio, L., First preliminary tests of the general relativistic gravitomagnetic field of the Sun and new constraints on a Yukawa-like fifth force from planetary data, *Planet. Space Sci.*, **55**(10), 1290-1298, 2007a (gr-qc/0507041).
- Iorio, L., Constraints on the range  $\lambda$  of Yukawa-like modifications to the Newtonian inverse-square law of gravitation from Solar System planetary motions, *J. High En. Phys.*, at press, arXiv:gr-qc/0708.1080v5, 2007b.
- Iorio, L., Dynamical determination of the mass of the Kuiper Belt from motions of the inner planets of the Solar system, *Mon. Not. Roy. Astron. Soc.*, **375**(4), 1311-1314, 2007c.
- Khriplovich, I.B., and Pitjeva, E.V., Upper Limits on Density of Dark Matter in Solar System, *Int. J. Mod. Phys. D*, **15**(4), 615-618, 2006.
- Khriplovich, I.B., Density of dark matter in Solar system and perihelion precession of planets, arXiv:astro-ph/0702260v2, 2007.
- Lense, J., and Thirring, H., Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie. *Phys. Z.*, **19**, 156-163, 1918. Translated and

- discussed by Mashhoon, B., Hehl F.W., and Theiss, D.S., On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, *Gen. Relativ. Gravit.*, **16**(8), 711-750, 1984. Reprinted in: Ruffini, R.J., Sigismondi, C. (Eds.), 2003. Nonlinear Gravitodynamics. World Scientific, Singapore, pp. 349-388.
- Lue, A., and Starkman, G., Gravitational leakage into extra dimensions: Probing dark energy using local gravity, *Phys. Rev. D*, **67**(6), 064002, 2003.
- Mena, O., Santiago, J., and Weller, J., Constraining Inverse-Curvature Gravity with Supernovae, *Phys. Rev. Lett.*, **96**(4), 041103, 2006.
- Milgrom, M., A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, *Astroph. J.*, **270**, 365-370, 1983.
- Navarro, I., and van Acoleyen, K., On the Newtonian limit of Generalized Modified Gravity Models, *Phys. Lett. B*, **622**(1-2), 1-5, 2005.
- Navarro, I., and van Acoleyen, K., Consistent long distance modification of gravity from inverse powers of the curvature, *J. Cosmol. Astropart. Phys.*, **3**, 8, 2006a.
- Navarro, I., and van Acoleyen, K., Modified gravity, dark energy and modified Newtonian dynamics, *J. Cosmol. Astropart. Phys.*, **9**, 6, 2006b.
- Nesseris, S., and Perivolaropoulos, L., Crossing the phantom divide: theoretical implications and observational status, *J. Cosmol. Astropart. Phys.*, **1**, 18, 2007a.
- Nesseris, S., and Perivolaropoulos, L., Limits of extended quintessence, *Phys. Rev. D*, **75**(2), 023517, 2007b.
- Nojiri, S., and Odintsov, S.D., Introduction to Modified Gravity and Gravitational Alternative for Dark Energy, *Int. J. Geom. Meth. Mod. Phys.*, **4**(1), 115-146, 2007.
- Pitjeva, E.V., Relativistic Effects and Solar Oblateness from Radar Observations of Planets and Spacecraft, *Astron. Lett.*, **31**(5), 340-349, 2005a.
- Pitjeva, E.V., High-Precision Ephemerides of Planets-EPM and Determination of Some Astronomical Constants, *Sol. Syst. Res.*, **39**(3), 176-186, 2005b.

- Ruggiero, M.L., and Iorio, L., Solar System planetary orbital motions and  $f(R)$  theories of gravity, *J. Cosmol. Astropart. Phys.*, **1**, 10, 2007.
- Sanders, R.H., and McGaugh, S.S., Modified Newtonian Dynamics as an Alternative to Dark Matter, *Ann. Rev. Astron. Astrophys.*, **40**, 263-317, 2002.
- Sanders, R.H., Solar system constraints on multifield theories of modified dynamics, *Mon. Not. Roy. Astron. Soc.*, **370**(3), 1519-1528, 2006.
- Sereno, M., and Jetzer, Ph., Dark matter versus modifications of the gravitational inverse-square law: results from planetary motion in the Solar system, *Mon. Not. Roy. Astron. Soc.*, **371**(2), 626-632, 2006a.
- Sereno, M., and Jetzer, Ph., Two-body problem with the cosmological constant and observational constraints, *Phys. Rev. D*, **73**(4), 044015, 2006b.
- Sereno, M., and Jetzer, Ph., Evolution of gravitational orbits in the expanding universe, *Phys. Rev. D*, **75**(6), 064031, 2007.
- Will, C.M., The Confrontation between General Relativity and Experiment, *Liv. Rev. Relativ.*, **9**(3), 2006.